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COMMON DOES NOT EQUAL EXCELLENT

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Foundation

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COMMON DOES NOT EQUAL EXCELLENT

1. EXECUTIVE SUMMARY

Common Does Not Equal Excellent addresses a number of issues related to the K-8 Common Core State Standards for Mathematics (CCSS-M). The CCSS-M is "common" in two senses of the word: It is shared among many adopting states, and it is mediocre. It may develop in students the ability to do common math, but analysis of the standards provides no compelling evidence that excellence in understanding or applying mathematics will be attained. This paper will take a look at some aspects of the CCSS-M that are of concern, although it is by no means an exhaustive examination of the standards. The examples used represent a limited sample of many similar issues contained within the standards. A brief overview of each section in this paper is provided below.

General Guidelines vs. Instructional Strategies

The Common Core State Standards for Mathematics document states: "These Standards do not dictate curriculum or teaching methods."[1] Examination of the CCSS-M finds much pedagogy embedded in the standards, contradicting this claim. For example, the CCSS-M includes this 3rd grade standard regarding fractions:

> Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

This standard is written with very specific directions on how to meet the standard. If the standard simply read, "represent fractions on the number line," the teacher could choose the curriculum and methods best suited to develop the skills needed to meet the standard. If not

[1] *Common Core State Standards for Mathematics*. National Governors Association Center for Best Practices, Council of Chief State School Officers Title: Common Core State Standards for Mathematics. Publisher: National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C. Copyright Date: 2010. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

directly dictating curriculum or teaching methods, the standards impose serious limitations on teachers' abilities to exercise professional judgment in determining curricular objectives and the relevant methods for use in their instructional delivery.

Embedding pedagogy into the standards blurs the line of distinction between standards and curriculum. In addition to mathematical topics, these standards, because of the way they are written and the embedded pedagogy, set standards for instruction. While lending themselves to standards-based instruction, they are, in contrast, instruction-based standards. The instructional approach is often prescribed and dictated by the standard, making them instruction-based standards.

The Meaning of Standard Algorithms

A definition of a mathematical algorithm is provided in this section along with examples of the standard algorithms for each of the basic math operations of addition, subtraction, multiplication, and division.

Alignment Criteria Limits Autonomy

The *K-8 Publishers' Criteria for the Common Core State Standards for Mathematics (Publishers' Criteria)*[2] is introduced in this section. The *Publishers' Criteria* was written by the lead CCSS-M writers to provide guidance for the alignment of textbooks and assessments. The criteria, and the aligned textbooks and assessments, developed will determine the instruction delivered to address the standards, not the best practices of teachers with evidence of past success. The instruction-based standards of the CCSS-M will force teachers to use less efficient methods rather than evidence-based best practices, such as mastery of the standard algorithms. This section also contends that publishing companies will follow the directives for alignment as put forth by the standards writers in the *Publishers' Criteria*.

Progression Toward Fluency with the Standard Algorithms

Progressions for the Common Core State Standards in Mathematics (Progressions)[3] is introduced in this section. The *Progressions* documents were written by the lead CCSS-M writers to provide direction for classroom instruction focusing on the strategies embedded in the standards (instruction-based standards). This section examines the emphasis on using the CCSS-M based instructional strategies while delaying the required use of the standard algorithms, and it provides a sample of one such strategy and how the Common Core-aligned testing consortia propose to assess it.

[2] *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*. National Governors Association, Council of Chief State School Officers, Achieve, Inc., Council of the Great City Schools, and National Association of State Boards of Education. July 7, 2012.
http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf

[3] *Progressions for the Common Core State Standards in Mathematics*.
http://ime.math.arizona.edu/progressions/

The CCSS-M requirement to use multiple strategies and alternative algorithms to solve computations is a chief complaint of parents who prefer that the standard algorithm be an accepted method. While the CCSS-M document doesn't contain language prohibiting its use before it is introduced in 4th grade, the lead standards writers make it very clear that the standard algorithms should not be taught earlier:

> The Standards distinguish strategies from algorithms. For example, students use strategies for addition and subtraction in Grades K-3, but are expected to fluently add and subtract whole numbers using standard algorithms by the end of Grade 4. The standard algorithms can be viewed as the culmination of a long progression of reasoning about quantities, the base-ten system, and the properties of operations.[4]

This section provides examples of how the *Progressions* documents promote specific classroom instruction focused on the strategies embedded in the standards (instruction-based standards) and cautions teachers against introducing the standard algorithm until 4th grade for addition and subtraction, 5th grade for multiplication, and 6th grade for division.

Past Influences on Publisher Alignments

There is a thread of continuity among the CCSS-M, the *Publishers' Criteria*, and the *Progressions* that goes beyond providing learning expectations. Since these documents, especially the *Progressions*, lay out expectations for textbook, assessment, and instructional alignment with the CCSS-M and go into greater detail about the meaning of strategies mentioned in the standards, it is apparent the standards writers intended to determine the instructional approach based on these standards.

Many of the instructional techniques and learning processes included in the CCSS-M and detailed in the *Publishers' Criteria* and *Progressions* documents are premised on past techniques and goals popular in reform math programs. This has given rise to a complaint that CCSS-M-aligned textbooks and materials promote reform, or "fuzzy math". Supporters of the CCSS-M have rejected this claim, stating that publishers and teachers have simply misinterpreted the standards. But it is clear that the publishing companies are following the directives set forth by the Standards Writing Team in the *Progressions* and *Publishers' Criteria* documents. Those responsible for writing the standards obviously have the clearest understanding of what an aligned curriculum looks like, and the interpretations of the standards by publishing companies appear well aligned with the standard writers' intent.

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[4] *K-5, Number and Operations in Base Ten. Progressions for the Common Core State Standards in Mathematics* (draft). The Common Core Standards Writing Team. April 21, 2012.
http://commoncoretools.me/wp-content/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf

Inadequate Preparation for Algebra I and Other Factors Limiting Student Success

This section brings out the importance of the standard algorithms and emphasizes how the delay in introducing them until 4th grade for addition and subtraction and 5th and 6th grade for multiplication and division, respectively, slows the progression towards, and may lead to, inadequate preparation for an authentic Algebra I course. Introducing, teaching, and having students practice the standard algorithms at earlier grade levels than called for in the CCSS-M is more likely to result in greater fluency with the standard algorithms. While it might seem novel to emphasize conceptual understanding and delay the requirement for the standard algorithms, there appears to be no empirical evidence that it will develop fluency or foster conceptual understanding.

Do High Cognitive-Demand Levels Equal Rigor in the CCSS-M?

This section presents how Norman Webb's Depth of Knowledge Levels[5] were used to require "higher-level thinking" in the CCSS-M without the prior development of prerequisite lower-level skills and concepts. The CCSS-M "rigor" requires students to perform at higher levels without adequately developing fundamental skills and concepts that provide the foundation for such higher-level performance. This kind of rigor places developmentally inappropriate expectations on students, especially at the lower grades. The "rigor" called for in the CCSS-M is not so much in the content as in how students are expected to display their knowledge.

Depth of Knowledge, CCSS-M Rigor, and High-Performing Countries

This section compares the types, or Depth of Knowledge Levels, of cognitive demands in the CCSS-M and those in the standards of higher-performing countries based on the percent of standards. A table showing a grade-by-grade, cognitive-demand comparison of the CCSS-M and Singapore standards reveals a notable difference: The CCSS-M has an emphasis on abstract-levels of cognitive demand, while higher-performing countries emphasize concrete-levels of cognitive demand. The CCSS-M has a higher percentage of standards emphasizing a demonstration of understanding and a lower percentage of standards emphasizing memorization and procedural fluency. Conversely, the standards of high-performing countries emphasize memorization and procedural fluency over the demonstration of understanding.

Topic Coherence and Focus in High-Performing Countries and the CCSS-M

This section compares topic coverage in the CCSS-M with the math standards of high-performing countries. The CCSS-M is not as coherent and focused as are those of high-performing countries.

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[5] Norman L. Webb and others. *Depth of Knowledge Levels*. "Web Alignment Tool". Wisconsin Center of Educational Research. University of Wisconsin-Madison. February 2, 2006
http://static.pdesas.org/content/documents/M1-Slide_19_DOK_Wheel_Slide.pdf

2. GENERAL GUIDELINES VS. INSTRUCTIONAL STRATEGIES

A recurring talking point of CCSS-M proponents is that the standards do not dictate how teachers must teach, but merely set forth what children should be learning in each grade.[6] However, a review of the CCSS-M shows that its standards do, in fact, repeatedly dictate instructional approaches. And if a curriculum is to be aligned with the CCSS, it must use these instructional approaches.

For example, the K-3 standards require students to solve addition and subtraction using strategies based on place value, "making ten (*e.g.*, 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (*e.g.*, 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); and by creating equivalent but easier or known sums (*e.g.*, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13)."[7] These strategies are required before students are taught the standard algorithms in 4th grade.

Similar "strategies based on place value" are then required for multiplication, such as the partial products method (*e.g.*, 324X6= (300X6)+(20X6)+(4X6)=1800+120+24= 1944), before introducing the standard algorithms in grade 5.[8] The same methods hold for division in 5th grade, with the standard algorithm not introduced until 6th grade.

To many, the CCSS-M are viewed as general guidelines enabling teachers to deliver standards-based instruction. Instead, as we will see, the CCSS-M includes many instruction-based standards. Instruction-based standards go beyond being general guidelines of the content and skills students need to master. Instead, they provide specific strategies on how the content and skills are to be taught and developed. In different terms, one might say the CCSS-M have embedded pedagogy.

These instruction-based standards dictate content and skills to be taught and developed in such a way as to delay the introduction and mastery of important skills. The development and delay of the standard algorithms for the basic math operations of addition, subtraction, multiplication, and division contribute to an inadequate preparation for algebra. To understand how this affects the student, we must first discuss the concept of the standard algorithms.

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[6] Compare that with the statement "Before Common Core State Standards we had standards, but rarely did we have instruction-based standards." From Student Achievement Partners, an entity founded by David Coleman and Jason Zimba, two architects of the Common Core, with associate Sue Pimental, a major Common Core author. *See* http://www.azed.gov/teacherprincipal-evaluation/files/2013/02/alberti-ccore-shifts-ppt.pdf

[7] *Common Core State Standards for Mathematics*.

[8] *Standard Algorithms in the Common Core State Standards: Developing and Understanding the Standard Algorithms in the Common Core State Standards*. Where's the Math? January 8, 2012. http://wheresthemath.com/math-standards/common-core-state-standards/standard-algorithms-in-the-common-core-state-standards-2/

3. THE STANDARD ALGORITHMS

Many CCSS-M standards call for the use of "strategies and algorithms based on place value" for several grades before being taught the standard algorithms for addition and subtraction in 4th grade, and multiplication and division in 5th and 6th grade, respectively. Granted, the standard algorithms are based on place value, but the wording of the standards requires the use of strategies and alternative algorithms typically less efficient than the standard algorithms. Knowing what an algorithm is and what the standard algorithms are is important for much of the discussion in this paper.

A mathematical algorithm is an ordered sequence of steps followed guaranteed to solve a specific class of math problems. An example of a class of problems would be adding numbers with any number of digits together or multiplying two numbers. To be considered an algorithm, the steps must always provide a correct answer for all problems in that class. A standard algorithm is the most efficient and universally practiced algorithm for a particular class.[9] Below are the standard algorithms with their related CCSS-M standards for the basic math operations of addition, subtraction, multiplication, and division of integers.

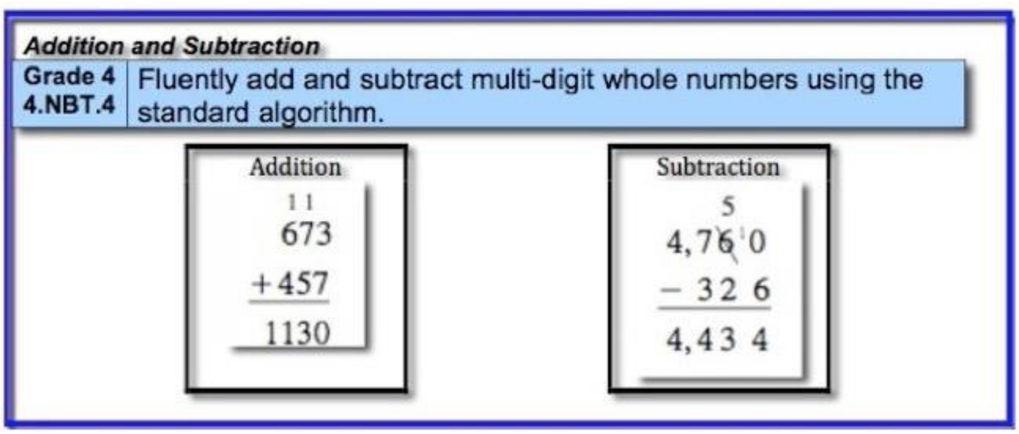

Figure 1.[10]

[9] Stanley Ocken. *Algorithms, Algebra, and Access.* Department of Mathematics, The City College of the City University of New York. September 2001.
https://app.box.com/s/qxjrj3ppz3yhd0a1cjvq

[10] Figure 1. *Standard Algorithms in the Common Core State Standards.*

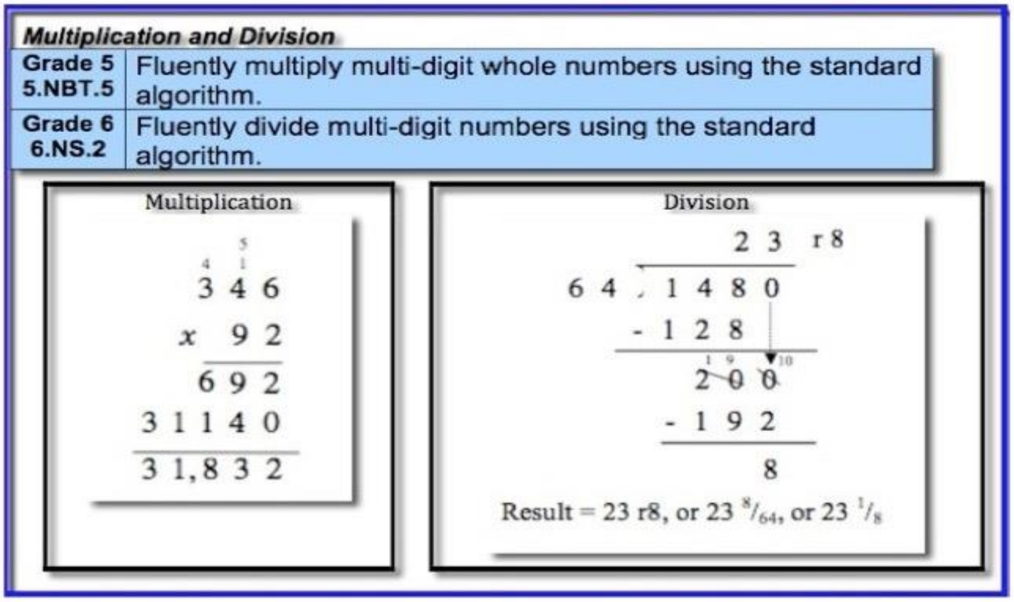

Figure 2.[11]

4. Alignment Criteria Limit Autonomy

Supporters of CCSS-M claim the standards are merely a floor and teachers are "free to go above and beyond the standards." However, this notion and the oft-used talking point about the standards' not dictating how teachers must teach both conflict with the *K-8 Publishers' Criteria for the Common Core State Standards for Mathematics*.[12] The *Publishers' Criteria* was written by the lead CCSS-M writers to inform textbook publishers and school districts on the requirements for full CCSS-M alignment. The *Publishers' Criteria* states that "in order to preserve the focus and coherence" of the CCSS-M, the consortia creating assessments aligned to the CCSS-M (the Smarter Balanced Assessment Consortium, or "SBAC," and the Partnership for Assessment of Readiness for College and Careers, or "PARCC") will not assess any standards that are not prescribed to a particular grade. In addition, the *Publishers' Criteria* provides guidance on what the "focus" standards will be for textbooks and the assessments. It specifies that unless approximately 75% of instructional time is devoted to these standards, the material is unlikely to be aligned. Adhering to such aligned materials is sure to keep teachers and schools in compliance with the standards.

Any standard or part of a standard is eligible to be included on assessments. If an instructional strategy is included in a standard, teachers can expect it to be assessed. Regardless of the long-term payoff of mastery of the standard algorithms, the short-term implications of student assessments will drive many school administrators to force teachers to stick to the CCSS-M and

[11] Figure 2. *Standard Algorithms in the Common Core State Standards.*
[12] *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*.

its idiosyncratic pedagogical expectations and prevent them from going above what is called for in the standards.

For example, CCSS-M standard 4.NBT.5 includes instructional strategies (shown in bold) for performing multiplication: "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, **using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."**[13]

In principle, the standard algorithms are also based on "strategies based on place value and properties of operations." In practice, however, the use of the standard algorithms in response to an item assessing this standard is likely to be marked "incomplete," because the strategies involved are not made explicit as is implied by the standard.

The figure below provides an example of how the above standard would be met utilizing the "area model" as prescribed in the above standard.

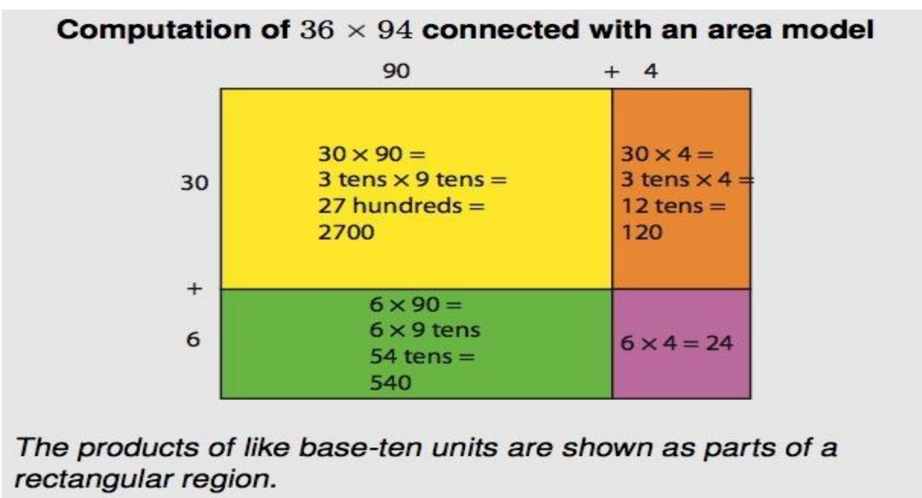

Figure 3.[14]

Below is a sample assessment item from the national Common Core assessment consortium SBAC, which requires the use of the area model instructional technique, stated in standard 4.NBT.5.

[13] *Common Core State Standards for Mathematics*.

[14] Figure 3. *K-5, Number and Operations in Base Ten. Progressions for the Common Core State Standards in Mathematics* (draft). The Common Core Standards Writing Team. April 21, 2012. http://commoncoretools.me/wpcontent/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf

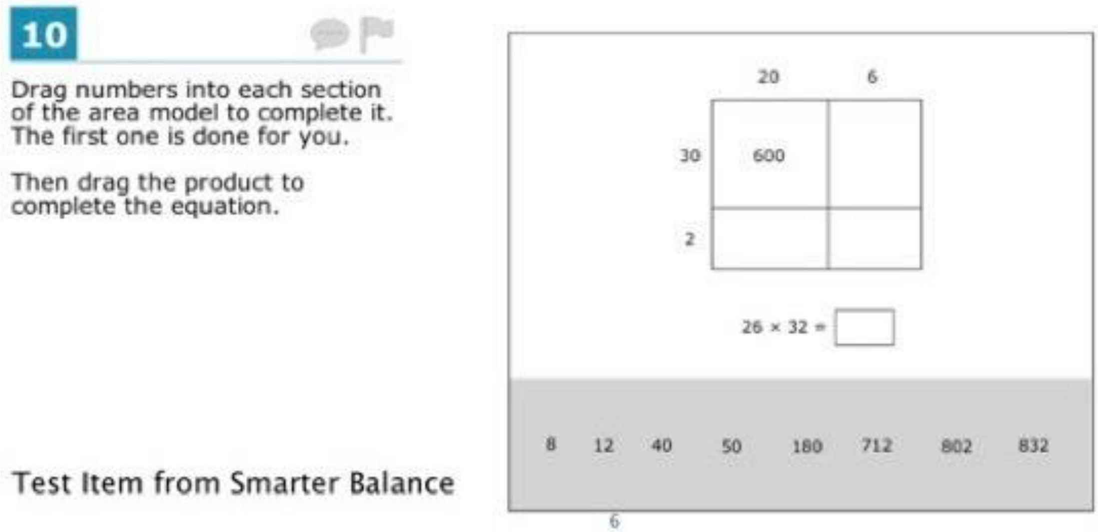

Figure 4.[15]

There is little doubt that regardless of a teacher's preference to use the instructional technique shown in Figure 3, popular in reform math programs, he or she must teach it in order for students to perform well on the Common Core high-stakes assessments as shown in Figure 4. With accountability measures tied to students' performance on the assessments, teachers will not be able to ignore the instructional strategies present in the standards.

5. Progression Towards Fluency with the Standard Algorithms

The *Publishers' Criteria* sets forth how students should progress towards fluent computation using the standard algorithms: "Progress towards these goals is interwoven with students developing conceptual understanding of the operations in question." Through a footnote, the *Publishers' Criteria* directs the reader to a second set of documents also written by the lead standards writers called *Progressions for the Common Core State Standards in Mathematics* (*Progressions*).[16] It provides further instruction on proper alignment with the CCSS-M and details the desired progressions for students learning the standard algorithm.

The *Progressions Front Matter*[17] document states: "The *Progressions* are intended to inform teacher preparation and professional development, curriculum organization, and textbook content. Thus, their audience includes teachers and anyone involved with schools, teacher education, test development, or curriculum development."[18]

[15] Figure 4. Smarter Balanced sample test item, grade 4, mathematics. http://sbac.portal.airast.org/practice-test/
[16] *Progressions for the Common Core State Standards in Mathematics*. http://ime.math.arizona.edu/progressions/
[17] *Front Matter for Progressions for the Common Core State Standards in Mathematics* (draft). The Common Core Standards Writing Team. July 2, 2013.http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_frontmatter_2013_07_30.pdf
[18] *Front Matter for Progressions.*

Progressions, the *Publishers' Criteria*, and the CCSS-M themselves direct teachers and textbook publishers to use instructional strategies embedded within the standards. *Progressions* is more detailed than the other documents and gives explicit directions for classroom instruction. It relies heavily on reform math strategies that encourage a delay in teaching the standard algorithms in favor of a conceptual emphasis, with the objective that children be able to conceptualize the "why" instead of the "how" in performing operations:

> The Standards distinguish strategies from algorithms. For example, students use strategies for addition and subtraction in Grades K-3, but are expected to fluently add and subtract whole numbers using standard algorithms by the end of Grade 4. Use of the standard algorithms can be viewed as the culmination of a long progression of reasoning about quantities, the base-ten system, and the properties of operations.[19]

For example, below is a CCSS-M standard for 1st grade addition and subtraction:

> 1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. **Use strategies such as counting on; making ten (e.g., 8 + 6 =8 +2 +4 = 10+4=14); decomposing a number leading to a ten (e.g., 13 – 4 = 13–3 – 1 = 10 – 1= 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8= 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).**[20] (Emphasis added)

Note that this is clearly more than a general guideline; a mere guideline would stop after the first sentence. It includes instructional strategies (shown in bold) as part of the standard, which requires them to be taught. These include counting on, making a ten, and decomposing. This, then, is an instruction-based standard. Further, the Standards Writing Team in *Progressions* provides clarity on exactly how students should represent their knowledge of this content. (*See* Figure 5 below.)

[19] *K-5, Number and Operations in Base Ten. Progressions for the Common Core State Standards in Mathematics* (draft). The Common Core Standards Writing Team. April 21, 2012. http://commoncoretools.me/wp-content/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf

[20] *Common Core State Standards for Mathematics*.

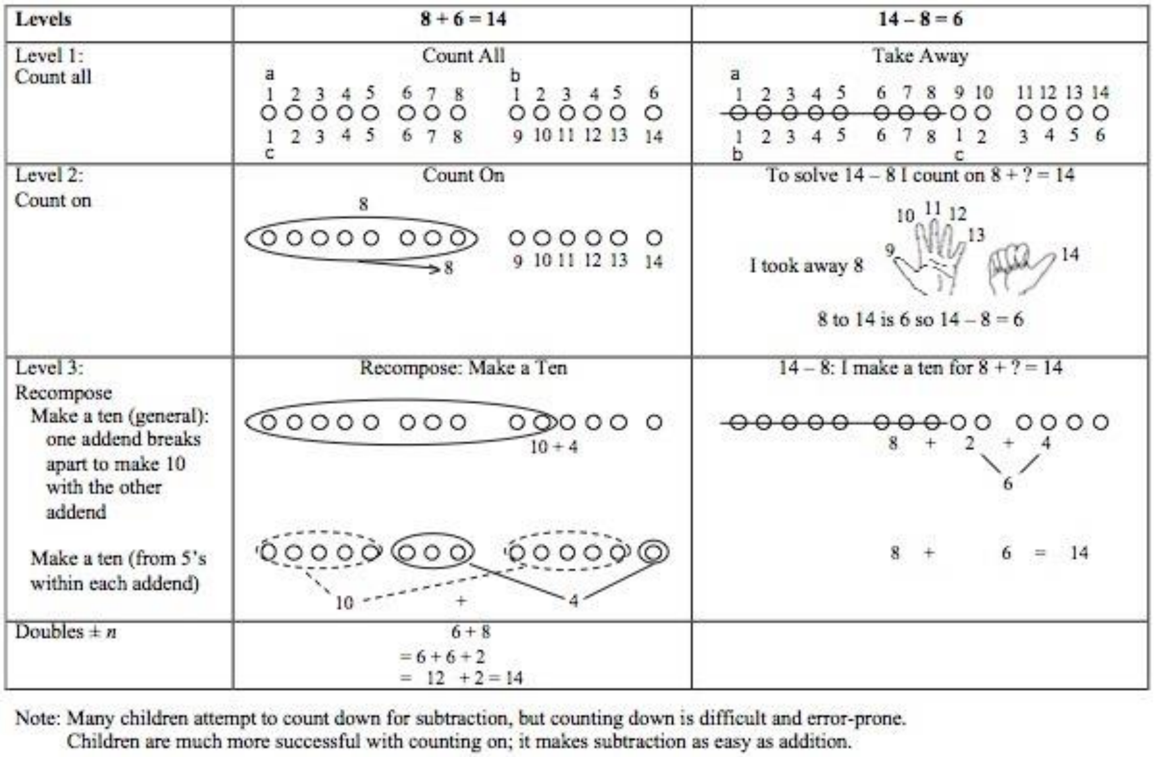

Figure 5.[21]

Figure 5 shows the CCSS-M intended to dictate methods for teaching the addition and subtraction standard 1.OA.6 and the different levels a student should use for performing addition and subtraction to meet the standard. That is how instruction-based standards are different from general guidelines. Note that it fails to offer or require the standard algorithm to be taught alongside it. The focus is on understanding the concept in multiple versions of its most simple forms before learning the procedure.

The instructional techniques remain consistent through 2nd grade to solve addition and subtraction problems, with the additional requirement of fluency using "mental strategies" rather than the standard algorithms. Thus, students progress with finger-counting and drawing pictures through the 2nd grade:

> 2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.[22] (See standard 1.OA.6 presented earlier for mental strategies)

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[21] K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. *Progressions for the Common Core State Standards in Mathematics (draft)*. The Common Core Standards Writing Team. May 29, 2011. http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

[22] *Common Core State Standards for Mathematics*.

Third grade calls for strategies and adds algorithms for addition and subtraction, but not the standard algorithms:

> 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.[23]

Although the two standards presented above call for fluency, the required fluency is not with the standard algorithm but with alternative algorithms. The Standards Writing Team explain this in *Progressions* as the build-up to students' learning the standard algorithms for addition and subtraction:

> Use place value understanding and properties of operations to perform multi-digit arithmetic. Students continue adding and subtracting within 1000. They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction. Such fluency can serve as preparation for learning standard algorithms in Grade 4, if the computational methods used can be connected with those algorithms.[24]

The CCSS-M delays the requirement for students to learn and use the standard algorithms for addition and subtraction until 4th grade. Unfortunately, the CCSS-M and *Progressions* include a similar delay in students' learning the standard algorithms for multiplication and division until 5th and 6th grade, respectively with standards 5.NBT.5 and 6.NS.2.

The *Progressions* document details how the CCSS-M standards for multiplication and division should be taught using the three levels prescribed for addition and subtraction: Level 1 – Count-on; Level 2 – Count-by; and Level 3 – Decompose:[25]

> Level 1 is making and counting all of the quantities involved in a multiplication or division computation. As before, the quantities can be represented by objects or with a diagram, but a diagram affords reflection and sharing when it is drawn on the board and explained by a student.[26]
>
> Level 2 is repeated counting on by a given number, such as for 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or

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[23] *Common Core State Standards for Mathematics*.
[24] *K-5, Number and Operations in Base Ten. Progressions for the Common Core State Standards in Mathematics* (draft).
[25] *K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. Progressions for the Common Core State Standards in Mathematics (draft).*
[26] *K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. Progressions for the Common Core State Standards in Mathematics (draft).*

> physical (*e.g.*, head bobs) pattern. For 8 x 3, you know the number of 3s and count by 3 until you reach 8 of them. For 24/3, you count by 3 until you hear 24, then look at your tracking method to see how many 3s you have.[27]
>
> Level 3 methods use the associative property or the distributive property to compose and decompose.… For example, students multiplicatively compose or decompose: 4 x 6 is easier to solve by counting by 3 eight times: 4 x 6 = 4 x (2 x 3) = (4 x 2) x 3 = 8 x 3. Students may know a product 1 or 2 ahead of or behind a given product and say "I know 6 x 5 is 30, so 7 x 5 is 30 + 5 more which is 35. This implicitly uses the distributive property: 7 x 5 = (6 + 1) x 5 = 6 x 5 + 1 x 5 = 30 + 5 = 35.[28]

Level 3 applies specifically to standard 3.OA.7, which calls for the use of strategies based on properties of operations to solve multiplication and division. "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations."[29]

The methods advanced by the Standards Writing Team in the *Progressions* document are the preferred instructional methods for multiplication and division through the 3rd grade. At grade four, the CCSS-M include the following standard:

> 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.[30]

The *Progressions* document gives more explicit detail on how to apply the area model technique and rectangular arrays to solve multiplication problems. The partial products algorithms, among others, are also introduced to perform multiplication and division. The standards and the *Progressions* document delay the standard algorithms for multiplication or division until 5th and 6th grades, respectively with standards 5.NBT.5 and 6.NS.2. Students using and practicing strategies and algorithms that cannot be generalized may develop difficult-to-change habits that may not work with the standard algorithm.

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[27] *K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. Progressions for the Common Core State Standards in Mathematics (draft).*
[28] *K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. Progressions for the Common Core State Standards in Mathematics (draft).*
[29] *Common Core State Standards for Mathematics.*
[30] *Common Core State Standards for Mathematics.*

The CCSS-M promote the idea of maximizing a student's conceptual understanding and reasoning skills, yet fail to acknowledge that both understanding and reasoning fundamentally depend on remembered content. Students who lack specific content knowledge developed through the memorization of facts and the practice of procedures will unfortunately rely on content-independent skills, such as "the count-by method" or "rectangular arrays" to solve computations.

It is curiously noted that the *Progressions* documents do not directly address teaching or teachers' delivery of instruction. Instead, the wording is couched in terms of what students are expected to do, use, or learn rather than what is to be taught. From this, it is uncertain if the authors expect students to develop their own skills and construct their own meaning, or if they intend for a teacher to provide some guidance, possibly even some explicit instruction. The avoidance of referring to teachers and instruction in the *Progressions* may be viewed as the Standards Writing Team's attempt to distance itself from criticism about the standards' clear dictation and prescription of curricula and teaching methods.

6. PAST INFLUENCES ON PUBLISHER ALIGNMENTS

In 1989, the National Council of Teachers of Mathematics released a set of math standards that have shaped the curriculum of most U.S. math education programs. The focus of these standards was similar to the CCSS-M, emphasizing the conceptual vs. the procedural and student-centered learning vs. direct instruction. Despite the warnings from mathematicians[31] who disagreed over the effectiveness of such approaches, publishing companies incorporated the NCTM standards into their textbooks. Referred to by many as reform or "fuzzy" math, the resulting math programs consistent with NCTM standards became the norm in many U.S. schools. The NCTM standards had five overarching goals for math education that, ironically, are mirrored in the CCSS-M Standards for Mathematical Practices. The NCTM goals were stated as:

> Learn to value mathematics: Students should have numerous, varied learning experiences that illuminate the cultural, historical, and scientific evolution of mathematics.
>
> Learn to reason mathematically: Skill in making conjectures, gathering evidence, and building an argument to support a theory are fundamental to doing mathematics. Therefore, sound reasoning should be valued as much as students' ability to find correct answers.

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[31] *An Open Letter to United States Secretary of Education, Richard Riley*
http://www.csun.edu/~vcmth00m/riley.html

> Learn to communicate mathematically: This goal is best accomplished in the context of problem solving that involves students in reading, writing, and talking in the language of mathematics. As students strive to communicate their ideas, they will learn to clarify, refine and consolidate their thinking.
>
> Become confident of their mathematical abilities: Study that relates to everyday life and builds students' sense of self-reliance will lead them to trust their thinking skills and apply their growing mathematical power.
>
> Become mathematical problem solvers.[32]

Many of the instructional techniques and learning processes included in the CCSS-M and detailed in the *Publishers' Criteria* and *Progressions* documents are premised on these NCTM goals and popular in reform math programs. This has given rise to a complaint that CCSS-M-aligned textbooks and materials promote reform, or "fuzzy," math. Supporters of the CCSS-M have rejected this claim, stating that publishers and teachers have simply misinterpreted the standards. But it is clear that the publishing companies are following the directives set forth by the Standards Writing Team in the *Progressions* and *Publishers' Criteria* documents. Those responsible for writing the standards obviously have the clearest understanding of what an aligned curriculum looks like, and the interpretations of the standards by publishing companies seem well aligned with the standards writers' intent.

In an era where research and data are supposed to be driving education policy, they are clearly ignored by the CCSS-M. With the stagnation of U.S. students' mathematics performance after decades of reform math and the dismal remediation rates at the college level, supporters of reform math practices should be conceding defeat. Instead, they claim that reform practices are sound, and blame the decades-long decline of math performance on poor implementation, among other excuses.

Unfortunately, we are already hearing the same "poor implementation" excuse in districts where the CCSS-M is not having the intended effect. CCSS-M supporters claim the standards are good, just poorly understood by teachers and misinterpreted by publishing companies. However, if America wants to be internationally competitive and improve math performance, a good first step would be to recognize that many reform math practices must finally be laid to rest.

[32] National Council of Teachers of Mathematics [NCTM], Commission on Standards for School Mathematics. (1989). *Curriculum and evaluation standards for school mathematics,* Reston, VA: Author.

7. Inadequate Preparation for Algebra I and Other Factors Limiting Student Success

The CCSS-M's use of strategies for performing operations keeps students in a multi-year holding pattern, slowing the progression towards Algebra I readiness. Stressing the concept over procedure, as the CCSS-M do, detracts from practicing and mastering the standard algorithms and other fundamental math skills before 8th grade, which leaves students who lack access to supplemental assistance unprepared to take Algebra I in 8th grade.

Preparation for a full Algebra I course by the beginning of 8th grade is critical for students who wish to reach Calculus in high school. Without completing Algebra I in 8th grade, students must take five years of math in four years. This is a doable feat for adept and motivated students, but one most students will choose to avoid. To increase the number of students reaching Calculus in high school, K-7 math standards should be structured to prepare all students for Algebra I. That will place more students on that path. However, some students will still need an additional year of preparation for Algebra I, and this paper does not advocate rushing students who don't possess the necessary foundations.

The standard algorithm for long division is considered an essential tool for success in algebra, yet the CCSS-M do not require fluency with it until the end of 6th grade. This delay leaves an inadequate amount of time for mastery if pre-algebra is to begin in 7th grade, which is necessary to reach Algebra I by the beginning of 8th grade. It also reduces the available time to master it and explore the nuance of different types of examples, such as divide by zero, or fractional division. According to math professors R. James Milgram and David Klein, this increases the likelihood that a student will be unsuccessful in higher math courses:

> Long division is a pre-skill that all students must master to automaticity for algebra (polynomial long division), pre-calculus (finding roots and asymptotes), and calculus (e.g., integration of rational functions and Laplace transforms.) Its demand for estimation and computation skills during the procedure develops number sense and facility with the decimal system of notation as no other single arithmetic operation affords.[33]

Stanley Ocken, Professor of Mathematics at City College New York, writes of the importance of the standard algorithms in *Algorithms, Algebra, and Access*.[34] In this paper he criticizes the National Council of Teachers of Mathematics (NCTM) Standards (1989, 2000) which (like the CCSS-M) de-emphasized the use of formal algorithms:

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[33] David Klein and R. James Milgram. *The Role of Long Division in the K-12 Curriculum*. http://www.csun.edu/%7Evcmth00m/longdivision.pdf

[34] Stanley Ocken.

> Equally important, practice with arithmetic algorithms is a student's first experience with the formal manipulation of mathematical symbols. Often lost in educators' attempts to help the child acquire "conceptual understanding" is the following basic reality: formal mathematical competency requires well-developed symbol manipulation skills.[35]

Students who do not develop accurate and efficient algebraic manipulations, he claims, "face virtually certain failure in any [college] math or physics course."[36]

The development of formal algorithmic skills was also supported by a 2001 report issued by a commission of math experts from City University of New York charged with reviewing New York City school system's math-education programs alignment to the NCTM Standards. Their report recommended the inclusion of "formal methods" and an "algorithmic approach" for later success in what are currently called STEM (science, technology, engineering, and math) fields. Although included in the public draft, the following recommendation was removed from the final report:

> It is important to recognize that the goals of the current New York State curriculum… come at a cost. Whenever the emphasis is placed on ensuring that applications are made to 'real world' situations….less emphasis is placed on arithmetical or mathematical ideas and the formal, abstract contextual settings sought particularly by students who will go on to become scientists, engineers, mathematicians, computer scientist, physicians, and educators of mathematics.[37]

Unfortunately, the dismissal of the importance of the standard algorithms continues with the CCSS-M. Despite the clear failure of similar past math-education standards, such as the NCTM Standards, the CCSS-M follow the same philosophies.

It is not only the inclusion of reform math strategies and delayed standard-algorithm fluency that will inadequately prepare students for a full Algebra I course in 8th grade. Students will also suffer from the exclusion of necessary key topics as identified in a white paper published by the Pioneer Institute, *Controlling Education From the Top*[38] in Exhibit B: Statement of Ze'ev Wurman Regarding Common Core Mathematic Standards. His statement presents many deficiencies of the CCSS-M, including the following in K-8 preparation:

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[35] Stanley Ocken.
[36] Stanley Ocken.
[37] Report of the panel on mathematics education in New York City schools, Board of Education of the City of New York, Press release N-141, May 30, 2001. As cited in Stanley Ocken. *Algorithms, Algebra, and Access*. Department of Mathematics, The City College of the City University of New York. September 2001. https://app.box.com/s/qxjrj3ppz3yhd0a1cjvq
[38] Emmett McGroarty and Jane Robbins. *Controlling Education From the Top: Why Common Core Is Bad for America.* A Pioneer Institute and American Principles Project White Paper, No. 87, May 2012.

> Common Core fails to teach prime factorization and consequently does not include teaching about least common denominators or greatest common factors.[39]
>
> Common Core fails to include conversions among fractions, decimals, and percents which was identified as a key skill by the National Research Council, the National Council of Teachers of Mathematics, and the presidential National Advisory Mathematics Panel.[40]
>
> Common Core starts teaching decimals only in grade 4, about two years behind the more rigorous state standards, and fails to use money as a natural introduction to this concept.[41]
>
> Common Core fails to teach in K-8 about key geometrical concepts such as the area of a triangle, sum of angles in a triangle, isosceles and equilateral triangles, or constructions with a straightedge and compass….[42]

8. Do High Cognitive-Demand Levels Equal Rigor in CCSS-M?

The CCSS-M is promoted as being more "rigorous" than previous sets of standards. The rigor, however, is not in more challenging mathematics; rather, it is in how students are asked to display their knowledge. This concept of rigor is based on the theory of Norman Webb from the Wisconsin Center of Educational Research, who developed a process and criteria for determining how standards, curricula, and assessments align with cognitive expectations. Webb's theory has four levels in which tasks are grouped together based on the "depth of knowledge" needed to complete each task:

> Level 1. Recall and Reproduction- calculate, define, draw, identify, label, illustrate, match, measure, memorize, recognize, repeat, recall, recite, state, tabulate, use, tell who-what-when-where-why.
>
> Level 2. Skills and Concepts - apply, categorize, determine cause and effect, classify, collect, compare, estimate, interpret, make observations, modify, solve, predict, graph, identify patterns, infer, interpret, make observations, modify.

[39] Emmett McGroarty and Jane Robbins, at Exhibit B, par. 5.
[40] Emmett McGroarty and Jane Robbins, at Exhibit B, par. 6.
[41] Emmett McGroarty and Jane Robbins, at Exhibit B, par. 8.4.
[42] Emmett McGroarty and Jane Robbins, at Exhibit B, par. 8.5.

> Level 3. Short-term Strategic Thinking - apprise, assess, cite, evidence, critique, develop a logical argument, differentiate, draw conclusions, explain phenomena in terms of concept, formulate, hypothesize, investigate, revise, use concepts to solve non-routine problems.
>
> Level 4. Extended Thinking - analyze, apply concepts, compose, connect, create, critique, defend, evaluate, judge, propose, prove, support, synthesize.[43] (*See Appendix A*)

The levels are based on the ideas of Bloom's taxonomy which originally labeled the levels as follows: Knowledge and Comprehension, Analysis and Application, Synthesis, and Evaluation.[44] (*See Appendix B*.) The levels were shown as a pyramid, with knowledge as the base.

The "rigor" that CCSS-M supporters refer to is based on the depth of knowledge called for by the CCSS-M: higher levels of cognitive demand equal more rigor. When proponents refer to "deeper learning" or "not being a mile wide and an inch deep" as (supposedly) were former standards, they are referring to the depth of knowledge. But the problem with CCSS-M is that it requires students to perform tasks at higher levels of the pyramid before acquiring a foundation of sufficient knowledge.

Under CCSS-M, the focus is not based on the knowledge of facts, and classroom time will therefore need to be devoted to teaching content not yet mastered by students. Were students taught the fundamental knowledge — including the standard algorithms — first, they could move quickly to higher levels of thinking. But CCSS-M's backwards approach will consume an exorbitant amount of classroom time teaching students how to do what they should have learned several years earlier. Instead of moving on from multiplying 2 two-digit numbers to multiplying 2 three-digit numbers, they are stuck in the quicksand of explaining and conjecture. Meanwhile, they are left in the dust by their peers in the higher-performing countries.

There is also a risk (or probability) that students will not master and automatically use the standard algorithms, which are the quickest, most efficient methods that work in every situation. Students may form a habit of using less efficient, cumbersome strategies and methods that are taught or self-formulated which do not always produce correct answers, causing an over-reliance on calculators.

It is useful to examine a particular standard in CCSS-M to understand the problem with this backwards approach and the developmental inappropriateness it engenders. Below is an

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[43] Norman L. Webb.

[44] Levels of Thinking in Bloom's Taxonomy and Webb's Depth of Knowledge.
http://www.stancoe.org/SCOE/iss/common_core/overview/overview_depth_of_knowledge/dok_bloom.pdf

example from the CCSS-M for 1st grade, which requires students, age six or seven, to solve an addition problem using deductive reasoning (Depth of Knowledge Level 3 to 4):

> 1.OA.3 Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)[45]

This is indeed rigorous, but is it appropriate to expect it from a student who has yet to memorize his math facts? The CCSS-M doesn't require memorization of addition facts until 2nd grade: "By end of Grade 2, know from memory all sums of two one-digit numbers." (2.OA.2)[46]

Concepts such as the commutative and associative properties are more easily understood and applied after mastery of both math facts and simple computations. In addition, widely accepted theories on childhood development conclude that many children at this age have yet to develop the cognitive structures to employ this type of thought process; their thinking is still too concrete.[47] If teachers are held accountable for students correctly answering this type of question on high-stakes assessments, a majority of classroom time will be spent drilling it into students, perhaps through the rote memorization of explanations for the concept tested. The teachers may eventually train students to answer an abstract question on an assessment, but they are unlikely to change the students' ability to internally understand the concept. To require this level of abstract thinking at age six or seven will produce frustrated children and nervous teachers.

Additionally, evaluating a student and teacher on "how" the student answers assessment questions regardless of a correct answer is fundamentally unfair. For example, if a student "computes" 12 x 12 = 144 using the standard algorithm, a "procedure" at Depth of Knowledge level 1-2, would he be deemed as not mastering the operation of multiplication unless he could strategically "formulate" a "differentiated" method using the rectangular array method at level 3? If a student can't put into words a written explanation for a correctly computed answer, does that mean he or she hasn't met the standards? How would this work for English Language Learners or analytical students who think symbolically? It doesn't. It will confuse and blur the data generated by assessments evaluating students' knowledge of math computations, because "how" they answer the problem is as important as getting the right answer.

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[45] *Common Core State Standards for Mathematics*.
[46] *Common Core State Standards for Mathematics*.
[47] Piaget, J., & Cook, M. T. (1952). *The Origins of Intelligence in Children*.

9. Depth of Knowledge, CCSS-M Rigor, and High-Performing Countries

The argument has been made that the CCSS-M and standards of high-performing countries are similarly coherent and focused. It is only natural that there is some alignment between topics, but the major difference is in the type of cognitive demand called for by the standards. In grades K-8, high-performing countries focus more on the first two levels of cognitive demand than do the CCSS-M. Professor Andrew Porter confirmed the disparity in math content between CCSS-M and the standards of high-performing countries in *Common Core Standards: The New Intended U.S. Curriculum*[48] published in *Educational Researcher*. He concluded:

> In mathematics, there are data for Finland, Japan, and Singapore on eighth-grade standards; alignments to the U.S. Common Core are .21, .17, and .13, respectively. All three of these countries have higher eighth-grade mathematics achievement levels than does the United States. The content differences that lead to these low levels of alignment for cognitive demand are, for all three countries, a much greater emphasis on "perform procedures" than found in the U.S. Common Core standards. For each country, approximately 75% of the content involves "perform procedures," whereas in the Common Core standards, the percentage for procedures is 38%.[49]

Is an early focus on higher depths of knowledge conducive to higher achievement in math? While we all want students to understand, subject-matter experts argue that understanding can grow only from knowledge of the procedural content, rather than from explicit teaching for depth. Hence, it is important to look at what actually works, rather than at what one hopes might work. While the CCSS-M writers have never provided a specific country to which the standards are benchmarked, there is little chance it was Singapore. In the CCSS-M, a greater percentage of standards emphasize "demonstrate understanding" (level 3) than "perform procedures" (level 2) or "memorize" (level 1). The table below shows that Singapore math standards have a greater percentage of their standards focused on "memorize" (level 1) and "perform procedures" (level 2) than does the CCSS-M for grades K-8.

Grade	Memorize	Perform Procedure	Demonstrate Understanding	Conjecture	Solve non-routine Problems
CCSS-M-1	7%	47%	47%	0%	0%

[48] Andrew Porter and others. *Common Core Standards: The New Intended U.S. Curriculum*. Published in Educational Researcher 40:103 (2011). May 5, 2011. http://iowaascd.org/files/8813/2543/8288/CommonCoreResearch010112.pdf

[49] Andrew Porter and others.

Singapore-1	29%	57%	12%	2%	0%
CCSS-M -2	11%	49%	32%	4%	4%
Singapore-2	24%	67%	7%	2%	0%
CCSS-M -3	7%	45%	33%	12%	4%
Singapore-3	24%	64%	11%	2%	0%
CCSS-M -4	14%	49%	29%	6%	3%
Singapore-4	14%	65%	11%	9%	2%
CCSS-M -5	8%	36%	34%	18%	4%
Singapore-5	18%	62%	8%	13%	0%
CCSS-M -6	8%	44%	42%	1%	5%
Singapore-6	6%	69%	20%	6%	0%
CCSS-M -7	6%	40%	36%	11%	8%
Singapore-7	10%	70%	12%	7%	1%
CCSS-M -8	15%	39%	27%	17%	2%
Singapore-8	16%	65%	10%	6%	2%
Average 1-8 CCSS-M	10%	44%	35%	6%	4%
Average 1-8 Singapore	18%	65%	11%	6%	1%

Figure 6.[50] Table reflects the percent of standards requiring the five depths of knowledge.

In contrast to the approach of the CCSS-M, high-performing countries, such as Singapore, concentrate on performing procedures, especially the standard algorithms. They do use pictorial or concrete examples to introduce a topic, but they do not expend valuable time having students continue with strategies and methods throughout several years. Instead, they continue advancing into more complex procedures, building to an authentic Algebra I course earlier than the CCSS-M. Singapore students focus and master the necessary number sense and skill with

[50] Figure 6. Data compiled from the totals provided in tables in this document. Brette Ashley Garner. *Internationally Benchmarked: Comparing the Common Core State Standards to the Singapore Mathematics Framework*. University of Texas at Austin, May 2013. http://repositories.lib.utexas.edu/bitstream/handle/2152/22441/GARNER-THESIS-2013.pdf?sequence=1

performing operations early on, with focused, coherent standards that build on strong foundations in arithmetic. This provides a quicker progression to Algebra I than the CCSS-M.

10. TOPIC COHERENCE AND FOCUS IN HIGH-PERFORMING COUNTRIES AND THE CCSS-M

The trend to keep more abstract content out of the early grades and focus on the fundamentals of arithmetic is prevalent in all top-performing countries, not just Singapore. Ze'ev Wurman compared topic coverage between the standards of high-performing countries on the international math assessment, Trends in International Math and Science Study (TIMSS), and that of the CCSS-M. Figure 7 shows the disparity he found. (*See Appendix D*)

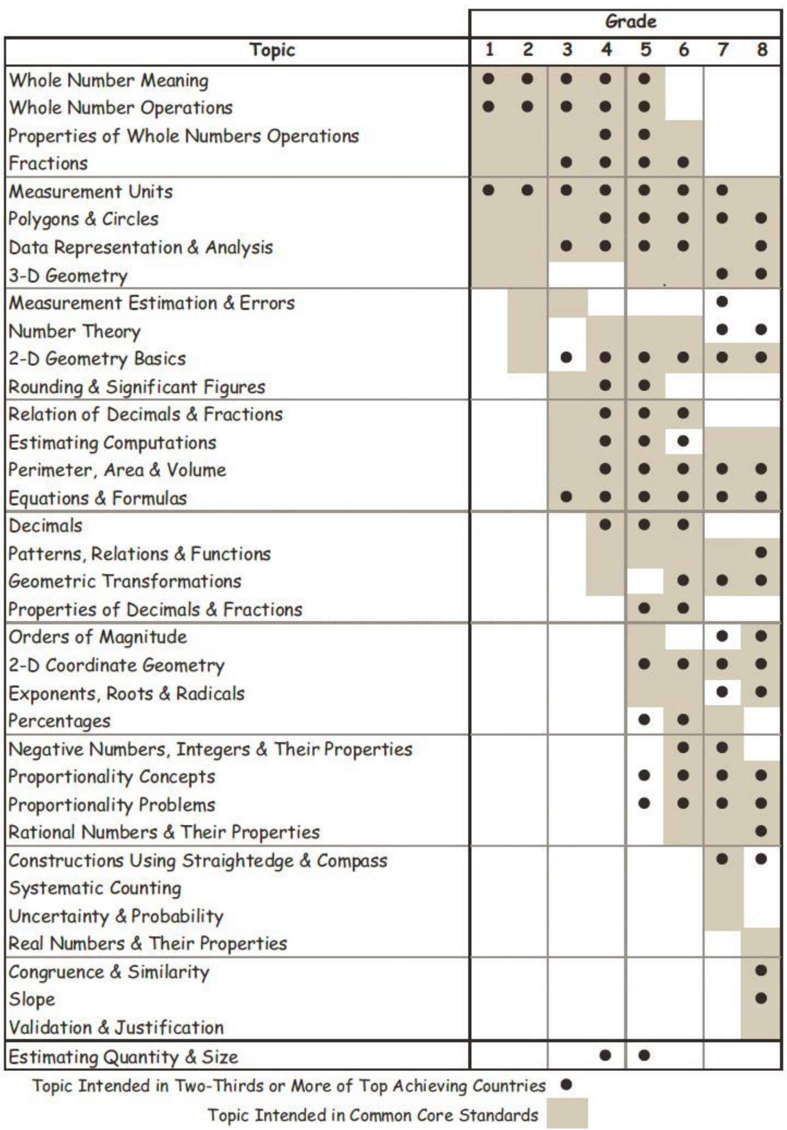

Figure 7.[51]

[51] Ze'ev Wurman. *Why Common Core's Math Standards Don't Measure Up*. Pioneer Institute. June 24, 2013.

High-performing countries cover three topics in grades 1-2: Whole Number Meaning, Whole Number Operations, and Measurement. By comparison, the CCSS-M are less focused and are "a mile-wide and an inch deep." They cover many more topics in grades 1-2: Whole Number Meaning, Whole Number Operations, Properties of Whole Number Operations, Fractions, Measurement, Polygons and Circles, Data Analysis, 3D Geometry, 2D Geometry Basics, Number Theory, and Measurement Estimation and Errors. The CCSS-M are neither coherent, focused, nor truly comparable to standards of high-performing countries.

The standards of top-performing countries reflect a coherence of topics. They are also concise and free of instructional strategies. *Appendix C: Comparison of Topic Coverage in Standards* shows the 1st-grade standards covering addition and subtraction from the CCSS-M, Singapore standards, and Massachusetts standards. The much longer CCSS-M are less clear than those of top-performing Singapore and Massachusetts, even in the earliest grades for the simplest of operations, and unlike those standards are packed with jargon. Starting in 1st grade, the addition and subtraction standards for Singapore and the top-performing state of Massachusetts both require the standard algorithms. In contrast, in the plethora of CCSS-M 1st-grade standards, the standard algorithms for addition and subtraction are absent and do not appear until 4th grade.

11. FINAL COMMENTS

There is a thread of continuity among the CCSS-M, *Publishers' Criteria*, and the *Progressions* documents. This is not surprising since they shared key writers. This thread of continuity goes beyond providing learning expectations. Since these documents, especially the *Progressions*, lay out expectations for textbook, assessment, and instructional alignment with the CCSS-M and go into greater detail about the meaning of strategies mentioned in the standards, they support the idea that the CCSS-M are instruction-based standards.

The U.S. is supposedly on a quest for global competitiveness, and proponents have billed the CCSS-M as the path to achieve it, despite the lack of evidence indicating the standards will increase students' achievement on international test scores. If we are to be competitive, it would seem logical to follow the example set by high-performing countries. Unfortunately, the Common Core State Standards appear to be leading us in an alternate direction, competing on a lower playing field.

There are simply too many fundamental differences between the CCSS-M and the standards and practices of high-performing countries to argue similarity between them. The expectation that these cognitively heavy but procedures-poor standards will improve student achievement has no empirical evidence to support it. Further, unlike the CCSS-M, the standards of high-

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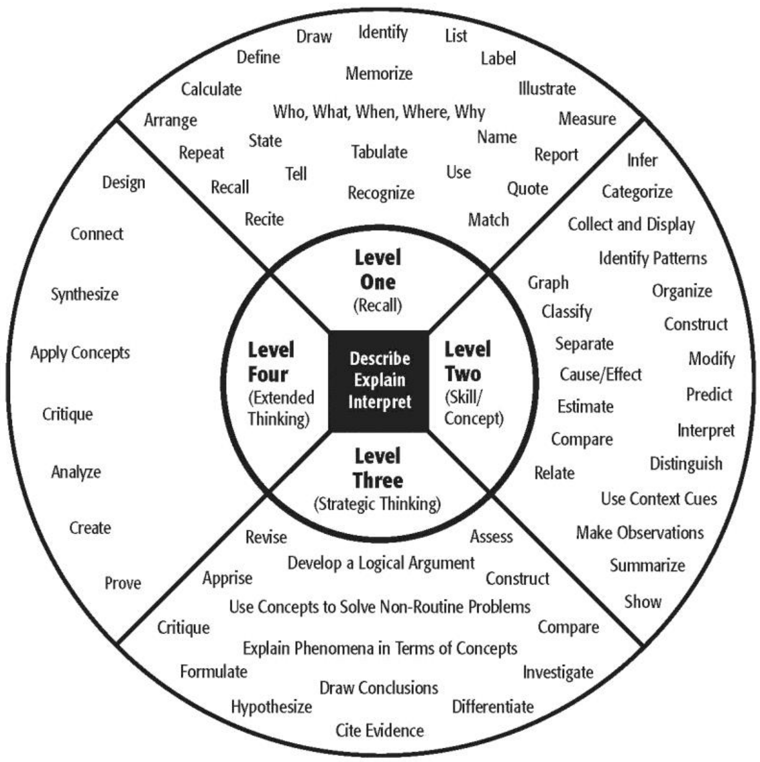
http://pioneerinstitute.org/news/why-common-cores-math-standards-dont-measure-up-by-guest-blogger-zeev-wurman/

performing countries are straightforward, clear, and concise. Those standards are free of embedded pedagogy and instructional mandates, and they do not delay the introduction and teaching of the standard algorithms and other critical skills. While the CCSS-M emphasize the conceptual over the procedural, high-performing countries take the opposite approach. Additionally, the topic coverage comparison shows the CCSS-M do not embody the coherence and focus of high-performing countries.

Of all the claims made by CCSS-M supporters, one thing is true: The CCSS-M have created a new path for math education in the United States. Sadly, this path is not based on evidence and has been well-worn by past reform-math initiatives that haven't led to a rise in student achievement or an increase in global competitiveness for the United States.

APPENDIX A: DEPTH OF KNOWLEDGE (DOK) LEVELS[1]

Depth of Knowledge (DOK) Levels

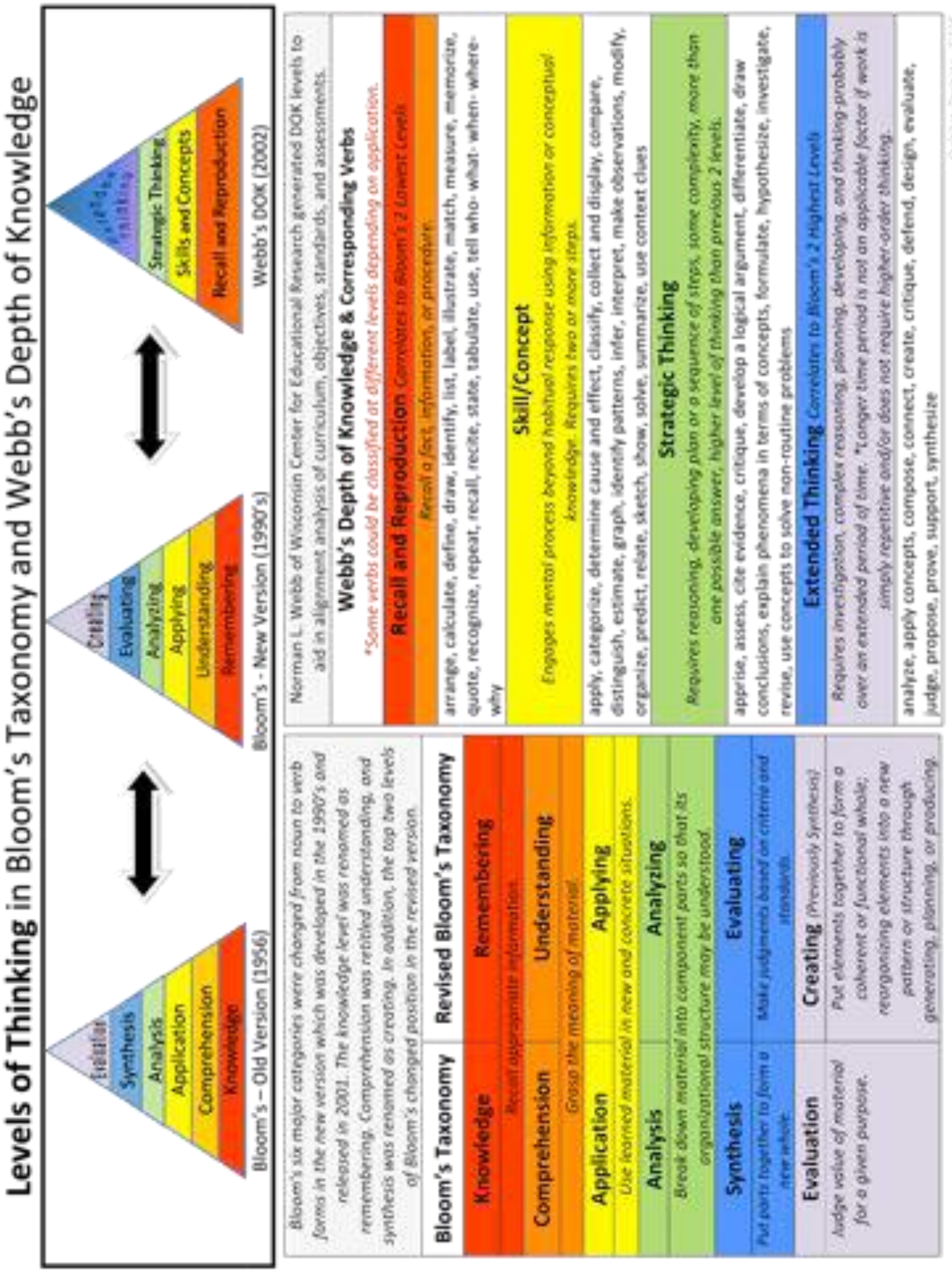


Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting. Conduct basic mathematical calculations. Label locations on a map. Represent in words or diagrams a scientific concept or relationship. Perform routine procedures like measuring length or using punctuation marks correctly. Describe the features of a place or people.	Identify and summarize the major events in a narrative. Use context cues to identify the meaning of unfamiliar words. Solve routine multiple-step problems. Describe the cause/effect of a particular event. Identify patterns in events or behavior. Formulate a routine problem given data and conditions. Organize, represent and interpret data.	Support ideas with details and examples. Use voice appropriate to the purpose and audience. Identify research questions and design investigations for a scientific problem. Develop a scientific model for a complex situation. Determine the author's purpose and describe how it affects the interpretation of a reading selection. Apply a concept in other contexts.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions. Apply mathematical model to illuminate a problem or situation. Analyze and synthesize information from multiple sources. Describe and illustrate how common themes are found across texts from different cultures. Design a mathematical model to inform and solve a practical or abstract situation.

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>

[1] Norman L. Webb and others. *Depth of Knowledge Levels*. "Web Alignment Tool". Wisconsin Center of Educational Research. University of Wisconsin-Madison. February 2, 2006 http://static.pdesas.org/content/documents/M1-Slide_19_DOK_Wheel_Slide.pdf

APPENDIX B: LEVELS OF THINKING IN BLOOM'S TAXONOMY AND WEBB'S DEPTH OF KNOWLEDGE[9]



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[2] Levels of Thinking in Bloom's Taxonomy and Webb's Depth of Knowledge.

Appendix C: Comparison of Topic Coverage in Standards

The standards of top-performing countries reflect a coherence of topics. They are also concise and free of any instructional strategies. Below are the first grade standards covering addition and subtraction from CCSS-M, Singapore, and Massachusetts. The CCSS-M are much longer and less clear than those of the top performers, even in the earliest grade for the most simple of operations.

Common Core State Standards for Mathematics Grade 1[3]
Addition and Subtraction

- CCSS.Math.Content.1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

- CCSS.Math.Content.1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

- CCSS.Math.Content.1.NBT.C.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

- CCSS.Math.Content.1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1]

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[3] *Common Core State Standards for Mathematics*. National Governors Association Center for Best Practices, Council of Chief State School Officers Title: Common Core State Standards for Mathematics. Publisher: National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C. Copyright Date: 2010. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

- CCSS.Math.Content.1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

- CCSS.Math.Content.1.OA.B.3 Apply properties of operations as strategies to add and subtract.[2] Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

- CCSS.Math.Content.1.OA.B.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

- CCSS.Math.Content.1.OA.C.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2)

- CCSS.Math.Content.1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13)

- CCSS.Math.Content.1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

- CCSS.Math.Content.1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.

These standards are neither clear nor concise and completely full of jargon. More importantly, they differ substantially from the standards used in high-performing countries. Take a look at the standards for first grade addition and subtraction in Singapore and the former standards of a top-performing U.S. state, Massachusetts. They are clear, concise and jargon free. They all require the standard algorithms for addition and subtraction. In contrast, in the plethora of CCSS-M first grade standards, the standard algorithm for addition and subtraction is absent and not required until fourth grade.

SINGAPORE GRADE 1 ADDITION AND SUBTRACTION[4]

- concepts of addition and subtraction,
- use of the addition symbol (+) or subtraction symbol (−) to write a mathematical statement for a given situation,
- comparing two numbers within 20 to tell how much one number is greater (or smaller) than the other,
- recognizing the relationship between addition and subtraction,
- building up the addition bonds up to 9 + 9 and committing to memory,
- solving 1-step word problems involving addition and subtraction within 20,
- addition of more than two 1-digit numbers,
- addition and subtraction within 100 involving, a 2-digit number and ones, a 2-digit number and tens, two 2-digit numbers,
- addition and subtraction using formal algorithms.

MASSACHUSETTS GRADE 1-2 BAND ADDITION AND SUBTRACTION[5]

- Demonstrate an understanding of various meanings of addition and subtraction, e.g., addition as combination (plus, combined with, more); subtraction as comparison (how much less, how much more), equalizing (how many more are needed to make these equal), and separation (how much remaining).
- Understand and use the inverse relationship between addition and subtraction (e.g., 8 + 6 = 14 is equivalent to 14 – 6 = 8 and is also equivalent to 14 – 8 = 6) to solve problems and check solutions.
- Know addition facts (addends to ten) and related subtraction facts, and use them to solve problems.
- Demonstrate the ability to add and subtract three-digit numbers accurately and efficiently.
- Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for addition (two 3-digit numbers and three 2-digit numbers) and subtraction (two 3-digit numbers).
- Estimate, calculate, and solve problems involving addition and subtraction of two-digit numbers. Describe differences between estimates and actual calculations.

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4 *Mathematics Syllabus Primary*. Curriculum Planning and Development, Ministry of Education Singapore. Copyright 2006. http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf

5 *Massachusetts Mathematics Curriculum Framework*. Massachusetts Department of Education. November 2000. http://www.doe.mass.edu/frameworks/math/2000/final.pdf

APPENDIX D: MATHEMATICS TOPICS AT EACH GRADE[6]

Mathematics topics intended at each grade by at least two thirds of the top- achieving countries.

	Grade							
Topic	1	2	3	4	5	6	7	8
Whole Number Meaning	●	●	●	●	●			
Whole Number Operations	●	●	●	●	●			
Measurement Units	●	●	●	●	●	●	●	
Fractions			●	●	●	●		
Equations & Formulas			●	●	●	●	●	●
Data Representation & Analysis			●	●	●	●		●
2-D Geometry Basics			●	●	●	●	●	●
Polygons & Circles				●	●	●	●	●
Perimeter, Area & Volume				●	●	●	●	●
Rounding & Significant Figures				●	●			
Estimating Computations				●	●	●		
Properties of Whole Numbers Operations				●	●			
Estimating Quantity & Size				●	●			
Decimals				●	●	●		
Relation of Decimals & Fractions				●	●	●		
Properties of Decimals & Fractions					●	●		
Percentages					●	●		
Proportionality Concepts					●	●	●	●
Proportionality Problems					●	●	●	●
2-D Coordinate Geometry					●	●	●	●
Geometric Transformations						●	●	●
Negative Numbers, Integers & Their Properties						●	●	
Number Theory							●	●
Exponents, Roots & Radicals							●	●
Orders of Magnitude							●	●
Measurement Estimation & Errors							●	
Constructions Using Straightedge & Compass							●	●
3-D Geometry							●	●
Congruence & Similarity								●
Rational Numbers & Their Properties								●
Functions								●
Slope								●

Intended by two-thirds or more of the top-achieving countries ●

6 Ze'ev Wurman. *Why Common Core's Math Standards Don't Measure Up*. Pioneer Institute. June 24, 2013. http://pioneerinstitute.org/news/why-common-cores-math-standards-dont-measure-up- by-guest-blogger-zeev-wurman/

Mathematics topics intended in the Common Core State Standards.

	Grade							
Topic	1	2	3	4	5	6	7	8
Whole Number Meaning	●	●	●	●	●			
Whole Number Operations	●	●	●	●	●			
Properties of Whole Numbers Operations	●	●	●	●	●	●		
Fractions	●	●	●	●	●	●		
Measurement Units	●	●	●	●	●	●	●	●
Polygons & Circles	●	●	●	●	●	●	●	●
Data Representation & Analysis	●	●	●	●	●	●	●	●
3-D Geometry	●	●			●	●	●	●
Measurement Estimation & Errors		●	●					
Number Theory		●		●	●	●		
2-D Geometry Basics		●		●	●	●	●	●
Rounding & Significant Figures			●	●	●			
Relation of Decimals & Fractions			●	●	●	●		
Estimating Computations			●	●	●		●	●
Perimeter, Area & Volume			●	●	●	●	●	●
Equations & Formulas			●	●	●	●	●	●
Decimals				●	●	●		
Patterns, Relations & Functions				●	●	●	●	●
Geometric Transformations				●		●	●	●
Properties of Decimals & Fractions					●	●		
Orders of Magnitude					●			●
2-D Coordinate Geometry					●	●	●	●
Exponents, Roots & Radicals					●	●		●
Percentages						●	●	
Negative Numbers, Integers & Their Properties						●	●	
Proportionality Concepts						●	●	●
Proportionality Problems						●	●	●
Rational Numbers & Their Properties						●	●	●
Constructions Using Straightedge & Compass							●	
Systematic Counting							●	
Uncertainty & Probability							●	
Real Numbers & Their Properties								●
Congruence & Similarity								●
Slope								●
Validation & Justification								●

Topic Intended in Common Core Standards ●

Mathematics topics intended at each grade in top-achieving countries compared to the Common Core State Standards. (The Common Core sequencing (gray shading) is not comparable to high-performing countries (black dots).

	Grade							
Topic	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Whole Number Meaning	●	●	●	●	●			
Whole Number Operations	●	●	●	●	●			
Properties of Whole Numbers Operations				●	●			
Fractions			●	●	●	●		
Measurement Units	●	●	●	●	●	●	●	
Polygons & Circles				●	●	●	●	●
Data Representation & Analysis			●	●	●	●		●
3-D Geometry							●	●
Measurement Estimation & Errors							●	
Number Theory							●	●
2-D Geometry Basics			●	●	●	●	●	●
Rounding & Significant Figures				●	●			
Relation of Decimals & Fractions				●	●	●		
Estimating Computations				●	●	●		
Perimeter, Area & Volume				●	●	●	●	●
Equations & Formulas			●	●	●	●	●	●
Decimals				●	●	●		
Patterns, Relations & Functions								●
Geometric Transformations						●	●	●
Properties of Decimals & Fractions					●	●		
Orders of Magnitude							●	●
2-D Coordinate Geometry					●	●	●	●
Exponents, Roots & Radicals							●	●
Percentages					●	●		
Negative Numbers, Integers & Their Properties						●	●	
Proportionality Concepts					●	●	●	●
Proportionality Problems					●	●	●	●
Rational Numbers & Their Properties								●
Constructions Using Straightedge & Compass							●	●
Systematic Counting								
Uncertainty & Probability								
Real Numbers & Their Properties								
Congruence & Similarity								●
Slope								●
Validation & Justification								
Estimating Quantity & Size				●	●			

Topic Intended in Two-Thirds or More of Top Achieving Countries ●

Topic Intended in Common Core Standards

About the Authors

Erin Tuttle is a citizen activist who led the effort to pass legislation which resulted in Indiana's being the first state to reject the Common Core Standards Initiative. Her efforts contributed to the national movement to reject federal intervention and restore the power of states and parents to govern education. She has written extensively on the deficiencies of the standards and provides research to other organizations fighting the Common Core Standards. A graduate of Indiana University, she worked in the broadcasting industry before devoting her full time to her family.

J.R. Wilson has 30 plus years' experience working in public education as an elementary classroom teacher, middle and high school math teacher, state department of education curriculum consultant, regional educational service agency staff development coordinator, and elementary principal. As a team member he has been involved in writing science and math standards. He has served on Where's the Math?'s Executive Committee. He participated in the U.S. Coalition for World Class Math's review of the Common Core State Standards Initiative's College and Career Readiness Standards for Mathematics draft, coordinated the Coalition's review of the March 10, 2010 draft of the Common Core State Standards for Mathematics, and participated in the Coalition's review of the final set of math standards.

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